THE UNIVERSITY OF WESTERN AUSTRALIA

SECOND SEMESTER EXAMINATIONS
NOVEMBER 1999

Robotics 315

231.315
This paper contains:
7 questions;
8 pages.

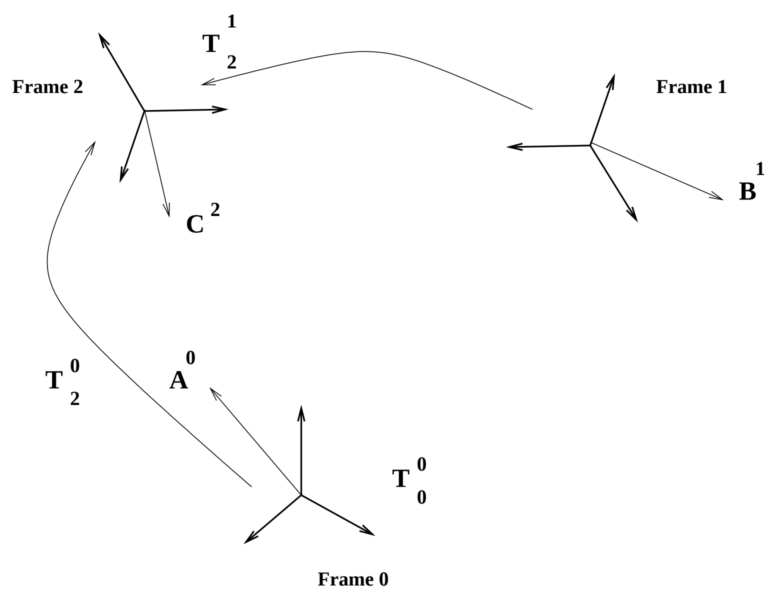
Time allowed: TWO HOURS

Reading time: TEN MINUTES

Candidates must answer Question 1 (which is worth 20 marks), and 5 of the remaining 6 questions (each worth 12 marks). The total number of marks is 80.

1.

Illustrated below are three coordinate frames and three positions defined with respect to these frames.



Assume you are given the following:

T_0^0 describing the base frame (frame 0).
T_2^0 describing frame 2 with respect to frame 0.
T_2^1 describing frame 2 with respect to frame 1.
A^0, point A described in terms of frame 0.
B^1, point B described in terms of frame 1.
C^2, point C described in terms of frame 2.

Write the expressions for the following:

(a) A in terms of frame 1.

(2)

(b) A in terms of frame 2.

(2)

(c) B in terms of frame 0.

(2)

(d) B in terms of frame 2.

(2)

(e) C in terms of frame 0.

(2)

(f) C in terms of frame 1.

(2)

(g) frame 0 in terms of frame 2.

(2)

(h) Explain the geometric interpretation of each column of a 4x4 homogeneous transformation matrix.

(3)

(i) Devise an algorithm that tests whether a matrix is a valid homogeneous transformation matrix.

(3)

2.

The following table gives the Denavit-Hartenberg parameters for a 3 link robot.

Joint	Variable	Angle	Length	Offset	Twist
1	angle	θ_1	1	1	$\pi/2$
2	angle	θ_2	0	-1	$-\pi/2$
3	offset	0	0	d_3	0

(a) Draw the configuration of the robot in its zero position, and when joint 1 has an angle of 0, joint 2 has an angle of $\pi/2$, and joint 3 has an offset of 1. For full marks you must draw the coordinate frames at each joint, and the end effector frame.

(6)

(b) If, at some configuration, you have the following data for the robot specified above:
$z_0, z_1, \text{and} z_2$ — the axes of the three joints,
$O_0, O_1, \text{and} O_2$ — the origins of the axis frames, and
P — the end effector location.

Write the expression for the $3x3$ Jacobian matrix for this robot.

(6)

3.

(a) Cubic polynomials are commonly used as interpolation functions for robot trajectories.

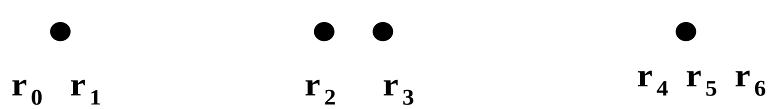
Calculate the coefficients of a cubic interpolation function $\Theta(t)$ that interpolates the motion of a joint from position θ_i at time $t = 0$, to position θ_f at time $t = 1$. The velocity of the joint at $t = 0$ must be 1 (not zero), and the velocity at $t = 1$ must be 0.

(7)

(b) Robot paths can be generated by interpolation in either joint-space or cartesian-space. Outline the advantages and disadvantages of interpolation in each of these spaces.

(3)

(c) Seven control points of a Bezier curve are shown below. Note that control points r_0 and r_1 are coincident, and that r_4, r_5 and r_6 are coincident. Assuming that one travels along the curve at constant parameter rate dt, sketch the approximate form of the velocity curve as a function of the curve parameter t, that would be produced by these points.



(2)

4.

(a) For any given robot configuration, and for each joint, there will be a direction of motion of the end-effector that maximizes the velocity that must be achieved by the joint to satisfy the required motion. How is the maximum joint velocity that may be required for a unit motion of the end effector calculated?

(3)

(b) For any joint of a robot, in some given configuration, there will be a set of directions of motion of the end-effector that result in that joint having a <u>zero velocity</u>. How would you identify this set of end effector motion directions?

(5)

(c) What is the equation that describes the relationship between forces applied at the end-effector, and torques/forces experienced at the joints of a stationary robot?

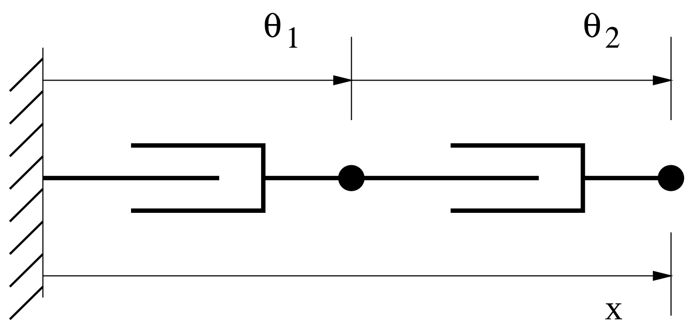
(2)

(d) Sketch a standard wrist mechanism consisting of three orthogonal rotary axes in series. Mark the regions of its workspace in which the inverse kinematics are ill-conditioned. Explain why the ill-conditioning arises.

(2)

5.

Consider the two link arm shown below. It consists of two prismatic joints joined in series to produce a kinematically redundant arm that can only move along the x axis.



The arm's motion rate control equation will have the following form

$$\left[\begin{array}{c} \dot{x} \end{array}\right] = \left[\begin{array}{cc} J_{11} & J_{12} \end{array}\right] . \left[\begin{array}{c} \dot{\theta}_1 \\ \dot{\theta}_2 \end{array}\right]$$

(a) Write the general form of the solution to the motion rate control equation for kinematically redundant robots. Explain each of the components that make up the solution.

(4)

(b) What property does a homogeneous solution of the equation above have? Give an example of a homogeneous solution for the robot above.

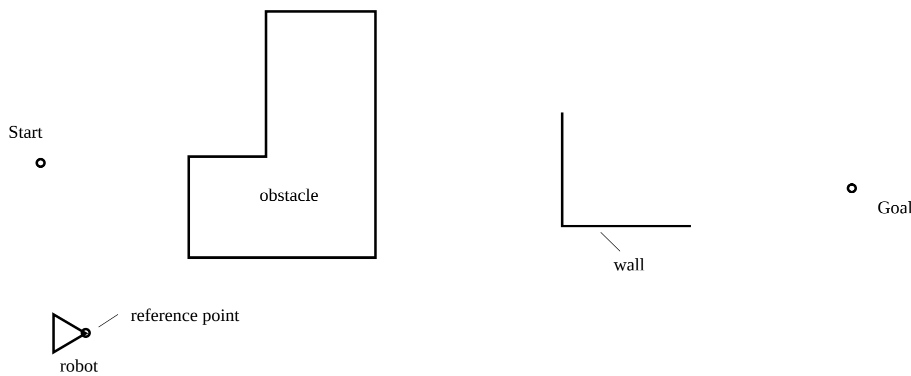
(2)

(c) Draw a diagram in joint velocity space that illustrates the construction of the components of the general solution to the motion rate control equation of the 2 link arm above.

(6)

6.

Duplicate the following diagram of a field of obstacles, and start and goal points, in your answer book.



(a) Draw how the obstacles should be grown in order to reduce the path planning problem for the robot to that of finding a collision free path for a point through a field of obstacles. Assume the robot is unable to rotate in its motion, it can only translate.

(3)

(b) Draw the visibility graph for your grown obstacles, and the start and goal points.

(2)

The potential field method for path planning requires two potential fields to be constructed.

(c) Write an expression that could be used to define an attractive field that pulls the robot towards the goal. Explain your equation, and sketch the shape of your function.

(2)

(d) Write an expression that constructs a field that could be used to repel the robot from obstacles. Explain your equation, and sketch the shape of your function.

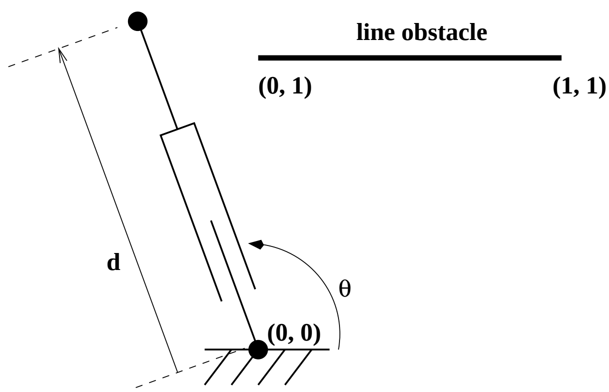
(3)

(e) What path planning difficulties can arise with the potential field method?

(2)

7.

Consider the 2 link planar arm shown below. The base of the robot is at $(0, 0)$. The first axis is a rotary joint with range $[0 - 2\pi]$. The second axis is a prismatic joint with a range of movement of [0.5–2].



(a) Write out the 2x2 Jacobian matrix relating the velocity of the end effector in x and y to the joint velocities $\dot{\theta}$ and $\dot{d}$. Each element of the Jacobian should be expressed in terms of the robot's current configuration (θ, d).

(5)

(b) What special property does the Jacobian matrix of this robot have?
What implications does this have for the kinematic solution of the arm?

(2)

(c) A line obstacle with endpoints at (0, 1) and (1, 1) is positioned in the workspace

Sketch the shape of this obstacle in the robot's configuration space. Your sketch can be approximate but the coordinates of the key points on the sketch must be clearly marked.

(5)