THE UNIVERSITY OF WESTERN AUSTRALIA

SECOND SEMESTER EXAMINATIONS
NOVEMBER 1998

Robotics 315

231.315
This paper contains:
8 questions;
8 pages.

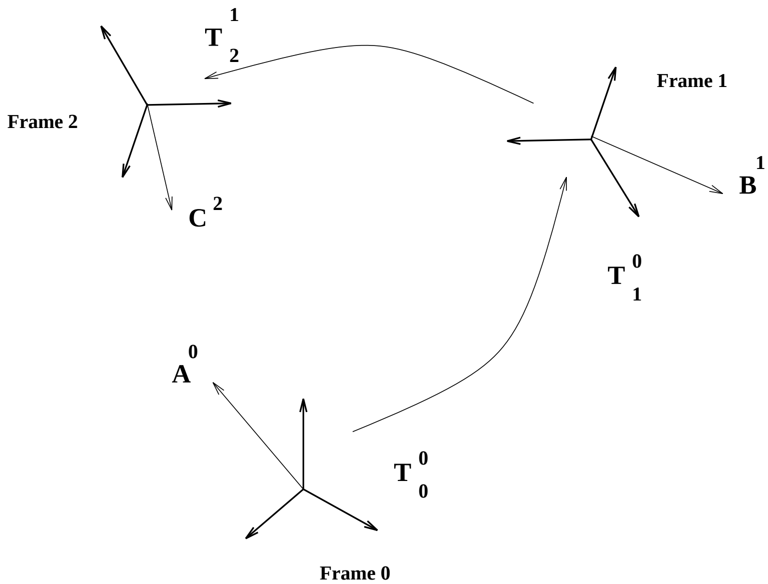
Time allowed: TWO HOURS

Reading time: TEN MINUTES

Candidates must answer Question 1 (which is worth 18 marks), and 6 of the remaining 7 questions (each worth 12 marks). The total number of marks is 90.

1.

Illustrated below are three coordinate frames and three positions defined with respect to these frames.



Assume you are given the following:

T_0^0 describing the base frame (frame 0).
T_1^0 describing frame 1 with respect to frame 0.
T_2^1 describing frame 2 with respect to frame 1.
A^0, point A described in terms of frame 0.
B^1, point B described in terms of frame 1.
C^2, point C described in terms of frame 2.

Write the expressions for the following:

(a) A in terms of frame 1.

(2)

(b) A in terms of frame 2.

(2)

(c) B in terms of frame 0.

(2)

(d) B in terms of frame 2.

(2)

(e) C in terms of frame 0.

(2)

(f) C in terms of frame 1.

(2)

(g) frame 0 in terms of frame 2.

(2)

(h) Devise an algorithm that tests whether a matrix is a valid homogeneous transformation matrix.

(4)

2.

The following table gives the Denavit-Hartenberg parameters for a 3 link robot.

Joint	Variable	Angle	Length	Offset	Twist
1	angle	θ	0	1	$\pi/2$
2	offset	$-\pi/2$	0	d	$\pi/2$
3	angle	θ	1	0	0

(a) Draw the configuration of the robot when joint 1 has an angle of 0, joint 2 has an offset of 1, and joint 3 has an angle of 0. For full marks you must draw the coordinate frames at each joint, and the end effector frame.

(6)

(b) If, at some configuration, you have the following:
$z_0, z_1, \text{and} z_2$ — the axes of the three joints,
$O_0, O_1, \text{and} O_2$ — the origins of the axis frames, and
P — the end effector location.

Write the expression for the $3x3$ Jacobian matrix for this robot.

(6)

3.

(a) Cubic polynomials are commonly used as interpolation functions for robot trajectories.

Calculate the coefficients of a cubic interpolation function $\Theta(t)$ that interpolates the motion of a joint from position θ_i at time $t = 0$, to position θ_f at time $t = 1$. The velocity of the joint at $t = 0$ and at $t = 1$ must be 0.

(7)

(b) Write the general expression for a Bezier polynomial curve with $n+1$ control points. Explain each component of the equation and define each variable you use.

(5)

4.

For any given robot configuration, and for each joint, there will be a direction of motion of the end-effector that maximizes the velocity that must be achieved by the joint to satisfy the required motion. This is the 'least favourable motion direction' for that joint.

(a) How is the least favourable motion direction determined for each joint?

(4)

(b) How is the maximum joint velocity that may be required for a unit motion of the end effector calculated?

(4)

The Jacobian matrix forms a mapping from joint velocity space to cartesian velocity space. For a 3 DOF robot we can think of all the possible unit joint velocity vectors as forming a sphere in joint velocity space.

(c) What will the mapping of this sphere look like in cartesian velocity space if the robot loses one degree of freedom?

(2)

(d) What will the mapping of this sphere look like in cartesian velocity space if the robot loses two degrees of freedom?

(2)

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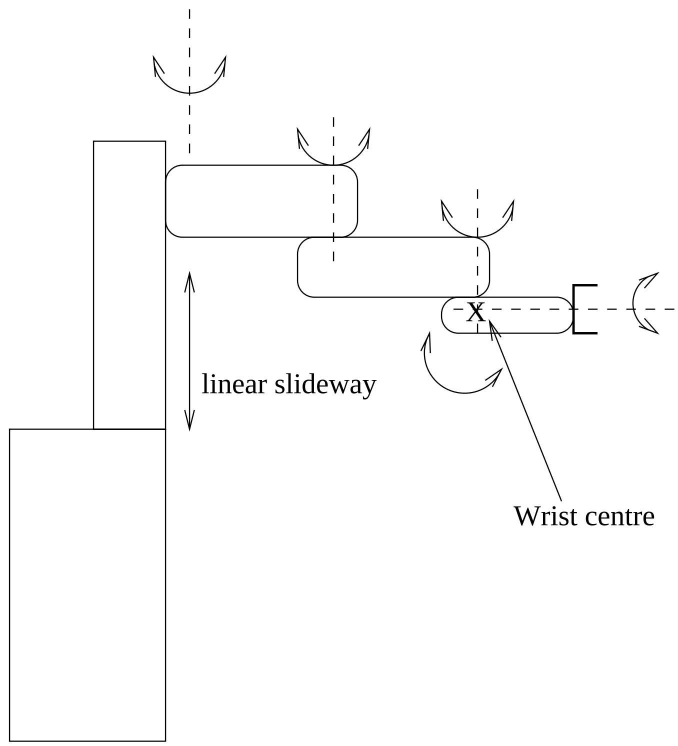
5. Shown below is a kinematic diagram of the RTX robot.



(a) In general, when the arm is not at a workspace boundary or at a singularity, how many kinematic solutions will the arm have? Describe briefly how each of the solutions will arise.

(2)

(b) Will the RTX have more, the same number, or fewer kinematic solutions than a Puma style 6 DOF arm? Why?

(2)

(c) Describe clearly all the steps in calculating the direct inverse kinematic solutions of the arm.

Assume you have all the Denavit-Hartenberg parameters of the arm, and that you have a function that inverts the Euler transform, and a function that calculates the inverse kinematic solutions of a two link rotary joint arm.

(8)

6.

A kinematically redundant 3 link planar arm will have a Jacobian matrix of the following form

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

(a) What property must a matrix G satisfy in order to be a generalized inverse of J?

(2)

(b) What property does a homogeneous solution of the equation above have?

(2)

(c) Write the general form of the solution to the motion rate control equation for kinematically redundant robots.

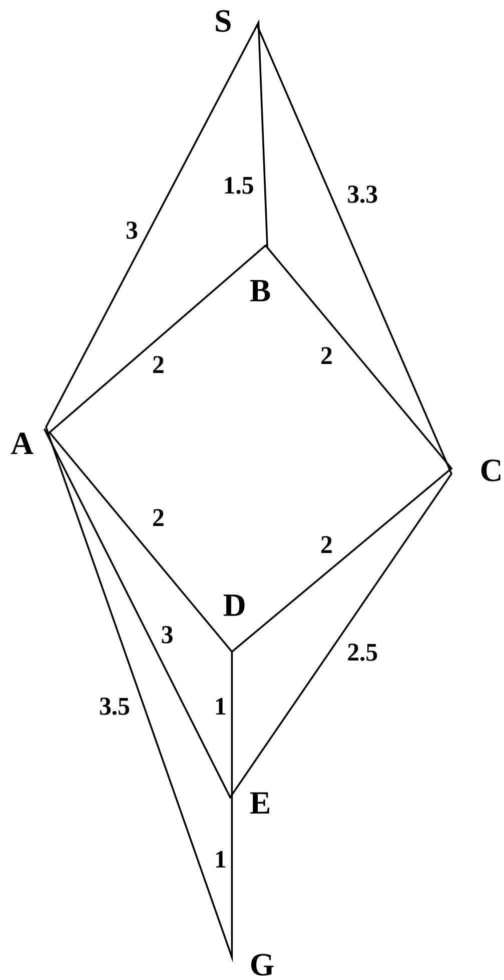
(4)

(d) Draw a diagram that illustrates the construction of the components of the general solution to the motion rate control equation of the 3 link arm above.

(4)

7.

Consider the visibility graph shown below.



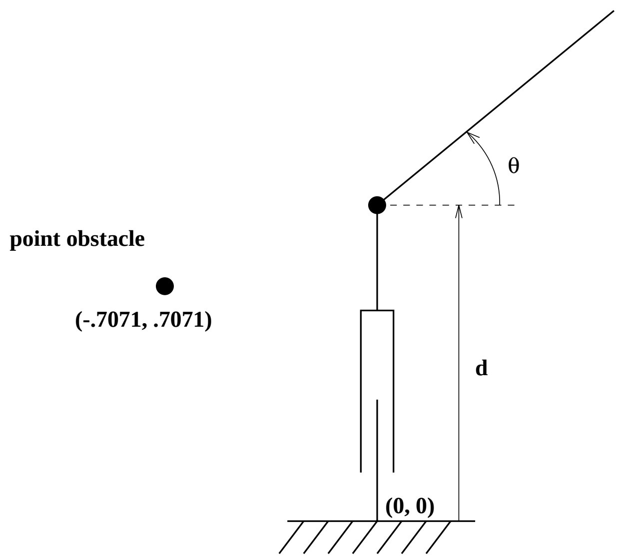
The costs of traveling between nodes are indicated on the graph. The direct distance from node B to the goal is 4.8 units, and the direct distance from node C to the goal is 3.1 units.

Find the shortest path from the start node, S, to the goal node, G, by applying the A^* algorithm. For full marks you must show what the queue of partial paths will be, and their estimated total costs, at each step of the algorithm.

(12)

8.

Consider the 2 link planar arm shown below. The base of the robot is at $(0, 0)$. The first axis is a vertical prismatic joint with a range of movement of $[0 - 2]$, the second axis is a rotary joint with range $[0 - 2\pi]$. The length of the second link is 1 unit.



(a) In what configurations will the kinematic solution of the arm be unique?

(2)

(b) In what configurations will the kinematic solution of the arm have multiple solutions?

(2)

(c) In what configurations will the kinematic solution of the arm be singular?

(2)

(d) A point obstacle is positioned at $(-1/\sqrt{2}, 1/\sqrt{2})$ (approximately (-.7071, .7071))

Sketch the shape of this obstacle in the robot's configuration space. Your sketch can be approximate but the coordinates of the key points on the sketch must be clearly marked.

(6)