THE UNIVERSITY OF WESTERN AUSTRALIA

SECOND SEMESTER EXAMINATIONS

NOVEMBER 1997

Robotics 315

231.315
This paper contains:
7 questions;
8 pages.

Time allowed: TWO HOURS

Reading time: TEN MINUTES

Marks for this paper total 100.
Candidates should attempt ALL Questions.

1.

(a) Explain the geometric interpretation of each column of a 4x4 homogeneous transformation matrix.

(4)

(b) Draw a diagram that clearly illustrates how the Denavit-Hartenburg parameters are used to describe the relative transformation from one link to the next at a **prismatic** joint of a serial manipulator.

(8)

(c) Write the sequence of homogeneous transformations that make up the Denavit-Hartenburg transformation matrix. Define the variables that you use in your expression.

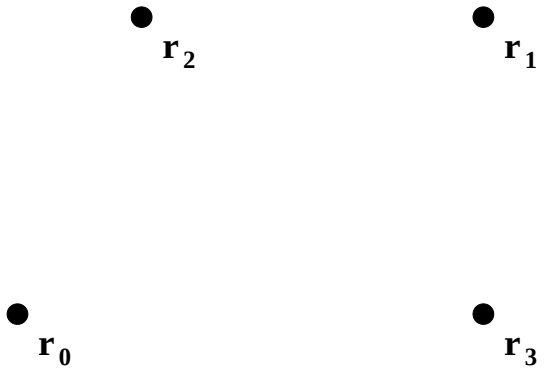
(4)

2.

(a) Write the general expression for a Bezier polynomial curve with $n+1$ control points. Explain each component of the equation and define each variable you use.

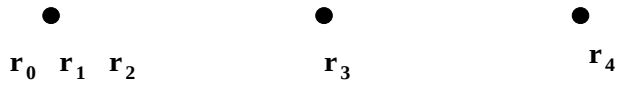
(6)

(b) Duplicate the control point arrangement shown below in your answer book. Sketch the Bezier curve that would be produced by these control points.



(2)

(c) Five control points are configured as shown below. Note that control points r_0, r_1 and r_2 have the same location. Assuming that one travels along the curve at constant parameter rate dt, sketch the approximate form of the velocity curve as a function of the curve parameter t, that would be produced by these control points.



(3)

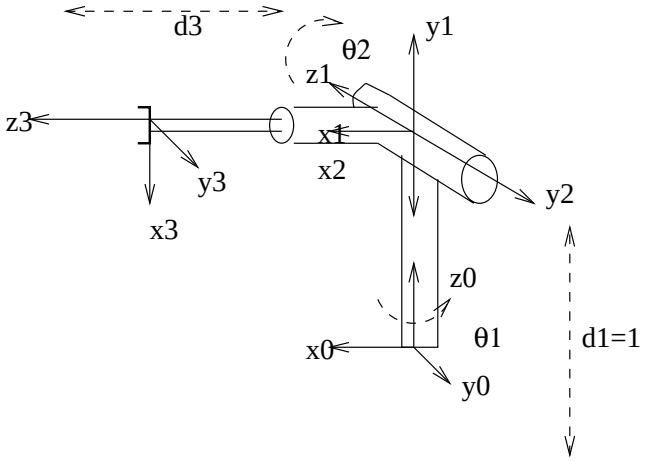
(d) How would you arrange the control points of two 3rd order Bezier curves so that the end of one curve was joined to the start of the second with continuity in both position and velocity? Use a sketch to illustrate your answer. (Note, a 3rd order Bezier curve is defined by 4 control points)

(4)

3. Drawn below is a 3 DOF Stanford arm. Joints 1 and 2 are rotary, joint 3 is prismatic. The Denavit-Hartenburg parmaters are as follows:

joint No	variable	angle	offset	length	twist
1	θ_1	θ_1	1	0	$\pi/2$
2	θ_2	θ_2	0	0	$-\pi/2$
3	d_3	0	d_3	0	0

In the robot's current configuration $\theta_1 = 0$, the variable offset d_3 is 1, and the x_1 axis is aligned with the z_2 axis.



(a) What is the value of θ_2?

(2)

(b) Write the 4x4 homogeneous transformation matrices that describe frames 1 and 3 in terms of the base frame for the robot in this configuration.

(6)

(c) Write the 3x3 Jacobian matrix for the robot in its current configuration.

(4)

(d) A force is applied at the end effector of the robot in this configuration. The force has a 2 Newton component in the x_0 direction, a 1 Newton in the y_0 direction, and a 3 Newton component in the z_0 direction. Assuming the robot is stationary what will the forces/torques in each joint be?

(5)

(e) Does this 3 DOF arm have any singularities? If so, in what configurations will they occur?

(2)

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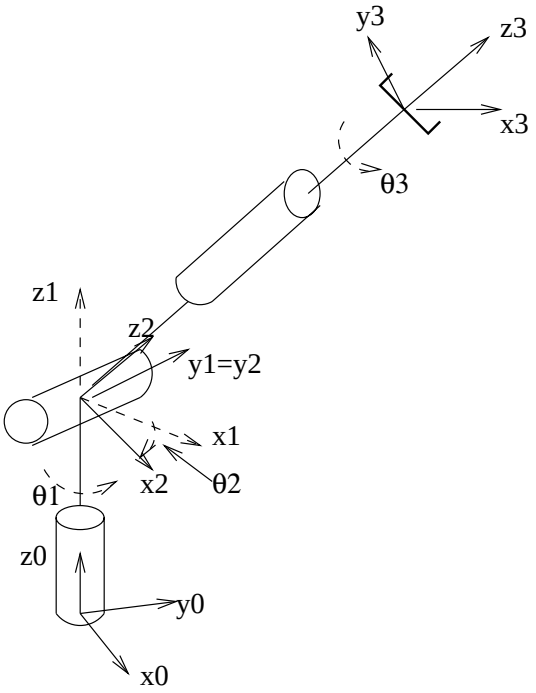
4. The orientation transformation generated by many robot wrist mechanisms corresponds to the Euler transform. That is, a rotation about the z axis, followed by a rotation about the y axis, followed by a rotation about the z axis again. This is shown below.



(a) Given the 3x3 rotation matrices:

$$rotz(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } roty(\theta) = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{bmatrix}$$

Write out the rotation matrices that describe frames 1 and 2 in terms of θ_1 and θ_2

(6)

(b) Write out the 3x3 Jacobian matrix that describes the end effector orientation velocity (described in terms of frame 0) with respect to the 3 wrist joint velocities.

(6)

(c) Write out the inverse of the Jacobian matrix when θ_2 is $\pi/2$.

(4)

5.

(a) A rotation transform R can be represented in terms of an equivalent rotation about some axis Ω by an angle θ.

Given two orientation frames, describe fully how you would calculate the equivalent axis of rotation and angle of rotation between the two frames using geometrical constructs.

(8)

(b) Assume you are given 3 different orientation frames; an initial frame, an intermediate frame and a final orientation frame. Why would it be difficult to interpolate orientation smoothly between all 3 frames?

(4)

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6. The potential field method for path planning requires two potential fields to be constructed.

(a) Write an expression that could be used to define an attractive field that pulls the robot towards the goal.

(5)

(b) Write an expression that constructs a field that could be used to repel the robot from obstacles.

(5)

In both cases above explain your equations, and sketch the shape of the potential field functions.

(c) What path planning difficulties can arise with the potential field method?

(2)

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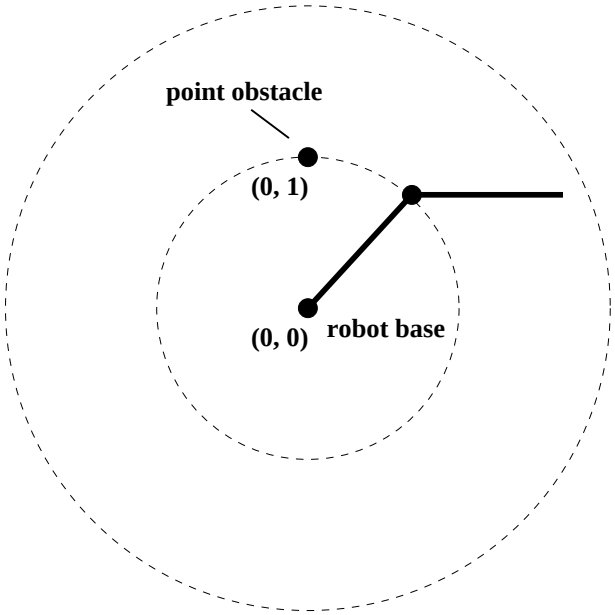
7. Drawn below is a 2 link planar arm with rotary joints.



The base of the robot is located at $(0, 0)$, both links have a length of 1 and the joints have no limits on their rotation. A point obstacle is located in the workspace at $(0, 1)$.

Sketch the shape of this point obstacle in the robot's configuration space. The sketch can be approximate, but the coordinates of the key points on the sketch must be marked clearly. Assume the joint angles are 0 when the robot is stretched out horizontally to the right.

(10)

Sample Solutions

1.

(a) (4)

(b) (8)

(c) (4)

2.

(a) (6)

(b) (2)

(c) (3)

(d) (4)

3.

(a) (2)

(b) (6)

(c) (4)

(d) (5)

(e) (2)

4.

(a) (6)

(b) (6)

(c) (4)

5.

(a) (8)

(b) (4)

6.

(a) (5)

(b) (5)

(c) (2)

7.

(10)