THE UNIVERSITY OF WESTERN AUSTRALIA

SECOND SEMESTER EXAMINATIONS

NOVEMBER 1996

Robotics 315

231.315
This paper contains:
8 questions;
7 pages.

Time allowed: TWO HOURS

Reading time: TEN MINUTES

Marks for this paper total 100.
Candidates should attempt ALL Questions.

1.

(a) A 4x4 homogeneous transformation matrix T can be represented as

$$T = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix}$$

where A is a 3x3 rotation matrix, b is a 3 element column vector and 0 is a zero 3 element row vector. Write an expression for the inverse of T in terms of A and b.

(3)

(b) Draw a diagram that clearly illustrates how the Denavit-Hartenburg parameters are used to describe the relative transformation from one link to the next at a rotary joint of a serial manipulator.

(8)

(c) Write the sequence of homogeneous transformations that make up the Denavit-Hartenburg transformation matrix. Define the variables that you use in your expression.

(4)

2.

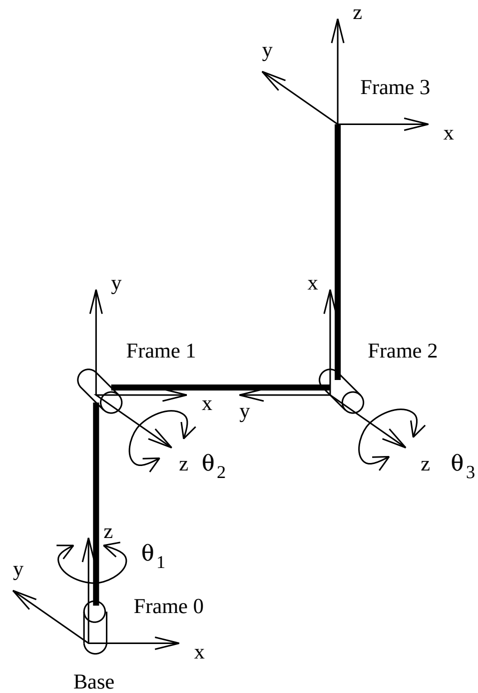
Cubic polynomials are commonly used as interpolation functions for robot trajectories.

Calculate the coefficients of a cubic interpolation function $\Theta(t)$ that interpolates the motion of a joint from position θ_i at time $t = 0$, to position θ_f at time $t = 1$. The velocity of the joint at $t = 0$ and at $t = 1$ is 0.

(7)

3.

Drawn below is a 3R robot with first and second joints set at 90 degrees. Each link is of length 1. Also shown are coordinate frames at each joint (the frames have been drawn slightly offset from the joint centres for clarity).



(a) Write the 4x4 homogeneous transformation matrices describing Frames 1, 2 and 3 in terms of Frame 0.

(6)

(b) Write the Jacobian matrix for the robot in its current configuration.

(7)

4.

In pseudo-code write the kinematic control algorithm that guides a robot along a trajectory using motion rate control. Assume you have a forward kinematics function that calculates the configuration of the robot and its Jacobian matrix, from the current set of joint positions. Note: Only consider control over the position of the end effector, not its orientation.

(6)

5.

For any given robot configuration, and for each joint, there will be a direction of motion of the end-effector that maximizes the velocity that must be achieved by the joint to satisfy the required motion. This is the 'least favourable motion direction' for that joint.

(a) How is the least favourable motion direction determined for each joint?

(6)

(b) How is the maximum joint velocity that may be required for a unit motion of the end effector calculated?

(5)

6.

A kinematically redundant 3 link planar arm will have a Jacobian matrix of the following form

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \end{bmatrix} . \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

(a) What property must a matrix G satisfy in order to be a generalized inverse of J?

(3)

(b) What property does a homogeneous solution of the equation above have?

(3)

(c) Write the general form of the solution to the motion rate control equation for kinematically redundant robots.

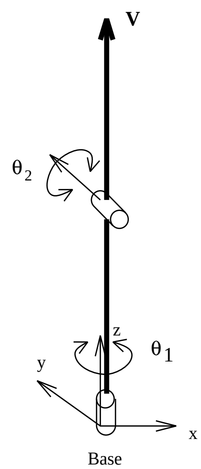
(5)

(d) Draw a diagram that illustrates the construction of the components of the general solution to the motion rate control equation of the 3 link arm above.

(6)

7.

Consider the mechanism shown below, designed to point in an arbitrary direction in 3D.



(a) Write down the forward kinematic map which, given θ_1 and θ_2, describes the pointing vector v (a unit vector in 3D) of the tip.

(6)

(b) Calculate the Jacobian matrix in terms of the joint values θ_1 and θ_2.

(7)

(c) What is the maximum rank of the Jacobian.

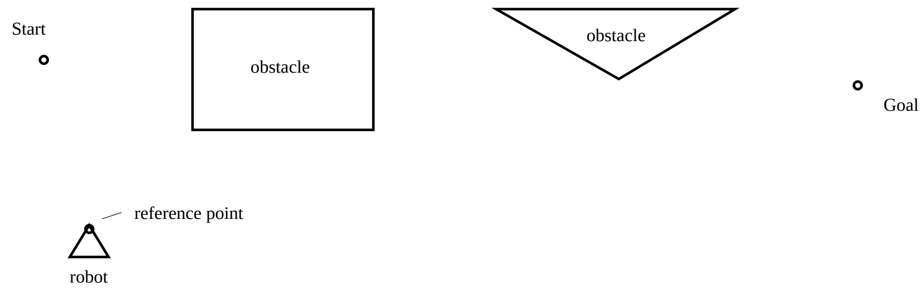
(2)

(d) In which configurations is the mechanism singular? Explain why.

(4)

8.

Duplicate the following diagram of a field of obstacles, and start and goal points, in your answer book.



(a) Draw how the obstacles should be grown in order to reduce the path planning problem for the robot to that of finding a collision free path for a point through a field of obstacles. Assume the robot is unable to rotate in its motion, it can only translate.

(6)

(b) Draw the visibility graph for your grown obstacles, and the start and goal points.

(6)

Sample Solutions

1.

(a) (3)

(b) (8)

(c) (4)

2.

(7)

3.

(a) (6)

(b) (7)

4.

(6)

5.

(a) (6)

(b) (5)

6.

(a) (3)

(b) (3)

(c) (5)

(d) (6)

7.

(a) (6)

(b) (7)

(c) (2)

(d) (4)

8.

(a) (6)

(b) (6)