THE UNIVERSITY OF WESTERN AUSTRALIA

SECOND SEMESTER EXAMINATIONS NOVEMBER 2000

DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING

Robotics (231.315)

This paper contains:
9 Pages
6 Questions

Time allowed: TWO HOURS
Reading time: TEN MINUTES

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Instructions

Candidates must answer FIVE questions to a total of 50 marks. Each question is worth 10 marks.

1.

A rigid body transformation is a mapping that preserves the orientation and distances between points. Such transformations in three dimensional space can be represented by 4×4 homogeneous transformations.

(a) What is the form of a 4×4 homogeneous transformation representing a rigid body motion?

(2)

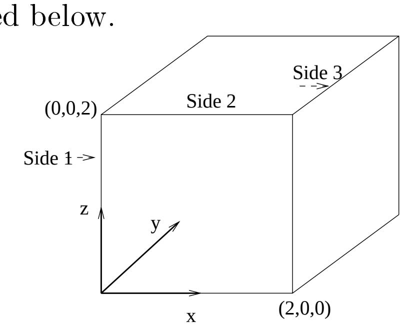
(b) How many degrees of freedom does a rigid body transformation have?

(1)

(c) Write down the homogeneous transformation describing a coordinate frame whose origin is at the point $(1, 2, 3)$, whose x-axis is in the base z direction, whose y-axis is in the base x direction, and whose z-axis is in the base y direction.

(3)

(d) Imagine a cube with sides of length 2, where three sides are aligned with the standard basis vectors, as illustrated below.



A base coordinate frame is placed at the origin, and travels along the edges of the cube by going up side 1, along side 2 and out along side 3. At each corner, the frame is rotated by 90^o so that the next transition is along the current z-axis.

Write down the homogeneous transformation that describes the position and orientation of the coordinate frame at the end of side 3 in base coordinates.

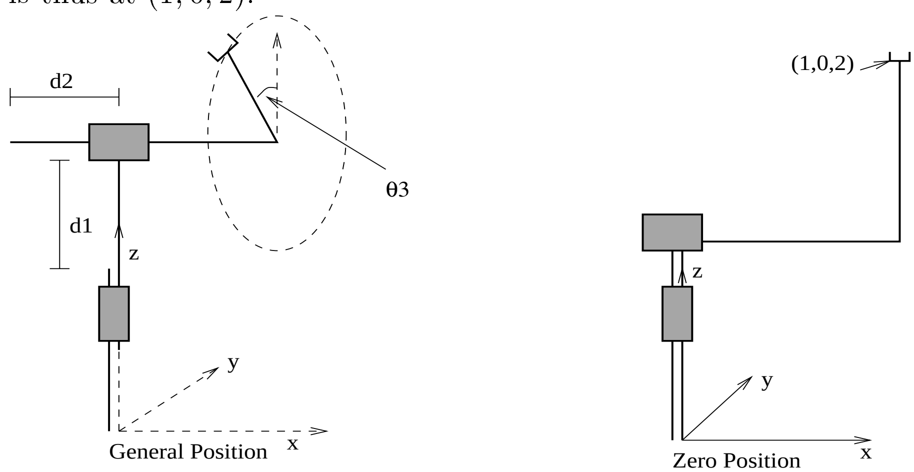
(4)

2.

(a) Explain what the *forward kinematics* of a robot are.

(1)

(b) Consider a 2P1R robot mechanism as illustrated below in a general position and in its zero position. Assume that all the linkages have physical length 1. The first link extends up the base z-axis, the second link extends parallel to the base x-axis, and the third link rotates in a plane that is parallel to the base $z-y$ plane. When the robot is in its zero position, the first link is not extended at all, the second link is fully extended along the x-axis, and the third link is pointing directly up the z-axis. In this zero position, the end effector is thus at $(1, 0, 2)$.



Work out the forward kinematics for this robot.

(4)

(c) Explain what it means for a robot to be in a *singular* configuration.

(1)

(d) Give an example of a 2-link planar robot in a singular configuration. Give an example of the same robot in an optimally conditioned configuration (that is, a configuration in which the Jacobian is simply a scalar multiple of the identity matrix).

(2)

(e) The Jacobian for the robot in part (b) is given by

$$J = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -\cos\theta \\ 1 & 0 & -\sin\theta \end{bmatrix}$$

Determine the configurations in which this robot is singular.

(2)

3.

(a) Define the *dextrous* workspace of a robot.

(1)

(b) Define the *reachable* workspace of a robot.

(1)

(c) Explain how the dextrous and reachable workspaces of a robot are related.

(1)

(d) Given a 2R planar robot arm with link lengths $l_1 = 2$ and $l_2 = 1$, illustrate the workspace of this robot and identify the dextrous and reachable parts of the workspace.

(3)

(e) Work out the inverse kinematics for the robot specified in part (d) above.

(4)

4.

(a) Define what is meant by a kinematically redundant arm.

(1)

(b) A 3R planar robot with link lengths $l_1 = l_2 = l_3 = 1$ is kinematically redundant with respect to the task of reaching a workspace coordinate (x, y).

Write down the forward kinematics for the 3R planar robot.

(2)

(c) Derive the Jacobian matrix for the 3R planar robot.

(2)

(d) Give a geometric interpretation for the solution of the motion rate control equations for a 3R planar robot. Draw a diagram illustrating the set of all solutions to $\dot{x} = J\dot{\theta}$, and all solutions to the homogeneous equation. In particular, mark the minimum norm solution to $\dot{x} = J\dot{\theta}$.

(3)

(e) Give an expression for calculating the pseudoinverse J^+ for the 3R planar robot. What are the dimensions of J^+?

(2)

5.

(a) Draw a diagram illustrating how coordinate frames are attached to a general robot linkage where all the joints are rotary. Your diagram should show frame $n-1$ attached to joint n, frame n attached to joint $n+1$, and all the transitions required to get from frame $n-1$ to frame n.

(3)

(b) Write down the general form of the Denavit Hartenberg matrix ${}^{n-1}_{n}T$ that describes the transformation from frame $n-1$ to frame n.

(2)

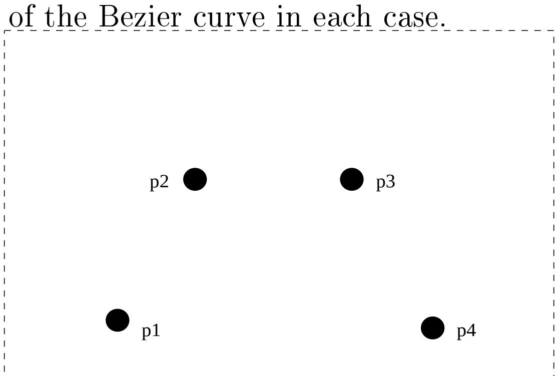
(c) Give the general expression for a Bezier polynomial curve with $n+1$ control points.

(1)

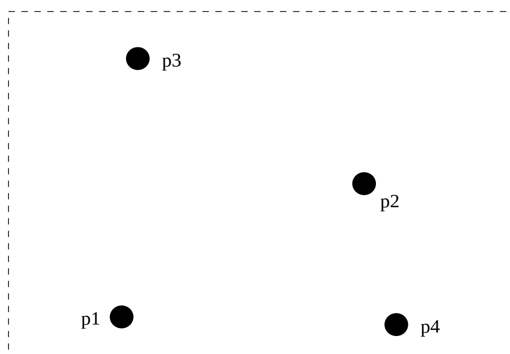
(d) In the case of a Bezier cubic polynomial, what are the known constraints at the start and end of the curve?

(2)

(e) Copy the following sets of control points into your exam booklet, and draw the characteristic polygons associated with each Bezier curve. Illustrate the general expected shape of the Bezier curve in each case.



CONTROL POINTS A

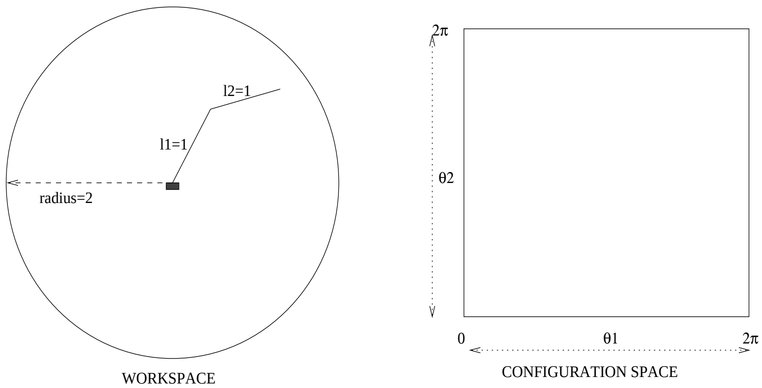


CONTROL POINTS B

(2)

6.

Consider a 2R planar robot with link lengths $l_1 = l_2 = 1$. With no actuator constraints, the reachable workspace of the tip of this robot is a disc of radius 2, and the configuration space of this robot is $[0, 2\pi] \times [0, 2\pi]$. Assume the zero configuration of this robot occurs when both links are aligned with the x-axis, and that movement of the second link is measured relative to the first.



(a) Draw a diagram illustrating the singular configurations in the configuration space of this robot.

(2)

(b) If the robot's physical workspace is restricted to the annulus defined by inner radius 1 and outer radius 2, illustrate on the configuration space the forbidden configurations of the robot.

(2)

(c) If the robot's workspace is now simply the inner disc of radius 1, illustrate on a separate diagram of the configuration space the forbidden configurations of the robot.

(2)

(d) Assume that the robot's movements are restricted to allow only the following configurations: $\theta_1 \in [\pi, 2\pi]$ and $\theta_2 \in [0, 2\pi]$. Draw a diagram showing the robot's workspace.

(2)

(e) Assume now that the robot's movements are restricted as follows: $\theta_1 \in [0, 2\pi]$ and $\theta_2 \in [\pi, 2\pi]$. Again illustrate the robot's workspace. Explain in words what these configuration restrictions mean in this case.

(2)